

# PROBING THE QCD VACUUM<sup>a</sup>

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Heavy quark bound states are used as significative probes of the QCD vacuum and the mechanism of confinement.

## 1 Strong interaction and topological nontrivial configurations

The focus of this school is on the “Topology of strongly correlated systems” in different areas ranging from QCD to condensed matter systems. In all these fields, investigations were presented about nontrivial topological configurations. In QCD such configurations are believed to be related to the nontrivial structure of the QCD vacuum and to the confinement mechanism<sup>1,2,3,4,5</sup> on one hand and to the breaking of the chiral symmetry<sup>6,3,7</sup> on the other hand. However, the QCD dynamics is extremely complicate and in order to test our understanding of the topological configurations, we need a systematic and under control way of parameterize the low energy physics in a frame that can still be simply related to the real experiments and/or to the lattice experiments<sup>9</sup>. It is the aim of my talk to show how this is possible and in which physical situations.

## 2 Heavy Quark Bound States

Heavy-quark bound systems are an ideal playground for theoretical ideas about strong QCD. In fact, while light quarks are highly “nonperturbative objects”, strongly interacting with the vacuum and with a mass almost entirely dominated by chiral symmetry breaking (topological nontrivial) effects, heavy quarks behave as “external sources” and thus as natural probes of the QCD vacuum. Being heavy at least the quark mass scale  $m$  can be treated perturbatively:  $m \gg \Lambda_{\text{QCD}}$ .  $\Lambda_{\text{QCD}}$  has here always to be understood as the scale at which nonperturbative effects become dominant. Furthermore, since the characteristic difference  $\Delta E$  between the energy levels is  $\Delta E \ll m$  inside such systems, they are nonrelativistic and can be described at leading order by the appropriate Schrödinger equation. This gives origin, as usual in nonrelativistic bound

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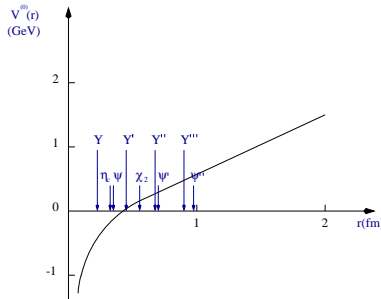


Figure 1:  $r_{Q\bar{Q}}$  vs Cornell potential (from Ref.<sup>8</sup>).

state, to several dynamical scales below  $m$ : the relative momentum  $p \sim mv$  (the inverse of which gives the characteristic spatial size  $r$  of the system), the energy  $E \sim mv^2$  (the inverse of which gives the characteristic time  $T$  of the system). Here,  $v$  is the velocity of the heavy quark in the bound state. In QCD,  $v$  is in general a function of the whole dynamics, perturbative and non-perturbative, but still remains a small parameter,  $v \ll 1$ , and thus the scales mentioned above turn out to be well separated and hierarchically ordered. It is well known that a technical (and quite hard) problem in bound state calculation is originated by the fact that such scales get entangled<sup>10</sup>. Thus, even in QED it turns out to be very useful to disentangle such scales. In QCD this becomes a deeper and more conceptual issue, since we need to separate as much as possible the perturbative contributions ('hard' physics at the high scale) from the nonperturbative effects ('soft or ultrasoft' physics at the low scale). *It is on these last contributions that we will eventually be in the position to use our knowledge about topologically nontrivial configurations.*

I will show here that in quarkonium physics the existence of different dynamical scales besides  $\Lambda_{\text{QCD}}$

- makes the calculations more complicate from a technical point of view;
- however, it makes also possible to test the non-perturbative nature of the QCD vacuum at different levels of deepness.

In fact, heavy quark bound states provide a full set of probes that may explore different characteristic distances, from the small to the large distance regime, and different dynamical situations, thus providing insights into perturbative/non-perturbative factorization mechanisms. In Fig. 1 we show a phenomenological determination of the characteristic radius  $r$  of heavy quarkonia against the phenomenological  $Q\bar{Q}$  potential. It is apparent that heavy quarkonia display a pattern of characteristic radii that extend from the perturbative

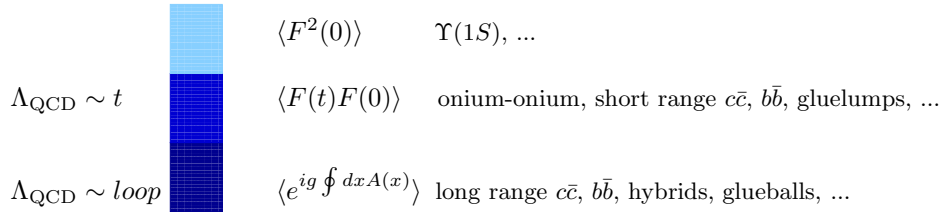


Figure 2: *The more extended the physical object, the more sensitive to non-perturbative physics.*

region  $r \ll \Lambda_{\text{QCD}}$  to the transition and the confinement region  $r \gg \Lambda_{\text{QCD}}$ . For ground state quarkonia (e.g.  $\Upsilon(1S)$ )  $r \ll 1/\Lambda_{\text{QCD}}$ : the scale  $1/r$  is perturbative (this also implies that the binding potential is perturbative). For most quarkonia  $r \sim 1/\Lambda_{\text{QCD}}$ : the scale  $1/r$  is nonperturbative. In order to be able to perform a systematic study, we need a clean and under control approach that allows us to factorize and resum the perturbative physics at the appropriate scale, leaving us with the appropriate parameterization of the nonperturbative physics at the low energy scale. This goal is achieved by constructing an effective field theory (EFT). The existence of a hierarchy of energy scales in quarkonium systems allows the construction of EFT with less and less dynamical degrees of freedom but completely equivalent to QCD. This leads ultimately to a field theory derived *quantum mechanical description of these systems*. We call potential nonrelativistic QCD (pNRQCD) the corresponding EFT. pNRQCD provides an unambiguous power counting in  $v$  which determines which operators are relevant at a given order of the expansion in  $v$ . We refer to<sup>11,10,12</sup> for details about pNRQCD. Here, we concentrate on the nonperturbative contributions: depending on the extension of the 'probe' systems such contributions enter parameterized in a completely different way. This is precisely the interesting point that may allow us to understand something about the QCD vacuum structure. In QCD we can never get completely rid of the nonperturbative effects. However, I will show that a pattern like the one represented in Fig 2 is realized in the heavy quark bound state nonperturbative dynamics. If the spatial and temporal characteristic scales of the bound system are perturbative (i.e. if  $mv \gg mv^2 \gg \Lambda_{\text{QCD}}$ ), then nonperturbative contributions arise only in the form of local gluon condensates which contribute to the energy levels. When the characteristic temporal scale starts to be dominated by  $\Lambda_{\text{QCD}}$  ( $mv^2 \sim \Lambda_{\text{QCD}}$ ), then the nonperturbative contributions are encoded in nonlocal (time-dependent) gluon condensates. As soon as  $1/\Lambda_{\text{QCD}}$  falls between the spatial and the temporal scale ( $mv > \Lambda_{\text{QCD}} > mv^2$ ), the nonperturbative nonlocal condensates enter the definition of the potential and give origin to short distance ( $r < 1/\Lambda_{\text{QCD}}$ ) nonperturbative corrections. At

the end, when  $\Lambda_{\text{QCD}}$  is comparable to the spatial size ( $mv \sim \Lambda_{\text{QCD}}$ ), the non-perturbative information is carried by bidimensional extended objects called Wilson loops. The potential is given in terms of them. Quite interestingly, the Wilson loop turns out to be the order parameter of confinement (in gluodynamics) and allows us to put in direct connection topological configurations and the heavy quark phenomenology.

As we see from Fig. 2, there are sufficient physical systems in nature to test all these dynamical situations.

### 3 Nonrelativistic quark bound states with a small characteristic radius: Condensates

For heavy quark bound systems for which the following hierarchy  $m \gg mv \gg mv^2 \gtrsim \Lambda_{\text{QCD}}$  holds, we can sequentially integrate out in perturbation theory both the hard scale  $m$  and the soft scale  $mv$ . Then  $v \sim \alpha_s$ . We denote by  $\mathbf{R} \equiv (\mathbf{x}_1 + \mathbf{x}_2)/2$  the center of mass of the  $Q\bar{Q}$  system and by  $\mathbf{r} \equiv \mathbf{x}_1 - \mathbf{x}_2$  the relative distance. Then at the scale of the 'matching'  $\mu'$  ( $mv \gg \mu' \gg mv^2, \Lambda_{\text{QCD}}$ ) we have as still dynamical degrees of freedom:  $Q\bar{Q}$  states with energy of order of the next relevant scale,  $O(\Lambda_{\text{QCD}}, mv^2)$ , momentum of order  $O(mv)$ , plus ultrasoft gluons  $A_\mu(\mathbf{R}, t)$  with energy and momentum of order  $O(\Lambda_{\text{QCD}}, mv^2)$ . Therefore only very low energy gluons remain in the EFT as dynamical degrees of freedom: they are *multipole expanded*. We find convenient to decompose the  $Q\bar{Q}$  state into a singlet  $S(\mathbf{R}, \mathbf{r}, t)$  and an octet  $O(\mathbf{R}, \mathbf{r}, t)$  with respect to color transformations. The pNRQCD Lagrangian is given at next to leading order (NLO) in the multipole expansion by<sup>11</sup>:

$$\begin{aligned} \mathcal{L} = & \text{Tr} \left\{ S^\dagger \left( i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s(r) - \sum_{n=1} \frac{V_s^{(n)}}{m^n} \right) S + O^\dagger \left( iD_0 - \frac{\mathbf{p}^2}{m} - V_o(r) - \sum_n \frac{V_o^{(n)}}{m^n} \right) O \right\} \\ & + gV_A(r) \text{Tr} \{ O^\dagger \mathbf{r} \cdot \mathbf{E} S + S^\dagger \mathbf{r} \cdot \mathbf{E} O \} + g \frac{V_B(r)}{2} \text{Tr} \{ O^\dagger \mathbf{r} \cdot \mathbf{E} O + O^\dagger O \mathbf{r} \cdot \mathbf{E} \} - \frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a}. \quad (1) \end{aligned}$$

All the gauge fields in Eq. (1) are evaluated in  $\mathbf{R}$  and  $t$ . In particular  $\mathbf{E} \equiv \mathbf{E}(\mathbf{R}, t)$  and  $iD_0 O \equiv i\partial_0 O - g[A_0(\mathbf{R}, t), O]$ . The  $V_j$  are potential matching coefficients: they are in the EFT the correspondent to the usual notion of the potential. We call  $V_s$  and  $V_o$  the singlet and octet static matching potentials respectively. The singlet sector of the Lagrangian gives rise to equations of motion of the Schrödinger type, while the last line of (1) contains retardation (or nonpotential) effects that start at the NLO in the multipole expansion. At this order the nonpotential effects come from the singlet-octet and octet-octet interaction mediated by an ultrasoft chromoelectric field. Recalling that  $\mathbf{r} \simeq 1/mv$  and that the operators count like the next relevant scale,  $O(mv^2, \Lambda_{\text{QCD}})$ , to the

power of the dimension, it follows that each term in the pNRQCD Lagrangian has a definite power counting. In the EFT language the potential is defined upon the integration of all the scales *up to the ultrasoft scale*  $mv^2$  and thus if  $\Lambda_{\text{QCD}} \lesssim mv^2$  the potential is given by a pure perturbative expansion in  $\alpha_s$  at all orders. The EFT gives us with (1) a precise prescription to calculate both potential and nonpotential contributions to the energy levels and to count their relevance in power of  $\alpha_s$ . Nonpotential effects start only at order  $\alpha_s^5 \ln \mu'$ . At this order (three loops) the static singlet potential starts to become sensitive to the infrared physics via a logarithmic dependence on  $\mu'^{11}$ . Such dependence is re-absorbed in a physical observable by non-potential contributions<sup>13</sup>. Non-perturbative effects are purely of nonpotential nature and the leading ones are carried by the chromoelectric field which mediates the singlet-octet interaction. From (1) the quarkonium energy levels have been calculated up to  $\alpha_s^5 \ln \mu'$  and are given by<sup>13</sup>

$$E_{n,l,j} = \langle nl | \frac{\mathbf{p}^2}{m} + V_s(\mu') + \frac{V_s^{(1)}(\mu')}{m} + \frac{V_s^{(2)}(\mu')}{m^2} | nl \rangle - i \frac{g^2}{3N_c} T_F \int_0^\infty dt \langle n, l | \mathbf{r} e^{it(E_n - H_o)} \mathbf{r} | n, l \rangle \langle \mathbf{E}^a(t) \phi(t, 0)_{ab}^{\text{adj}} \mathbf{E}^b(0) \rangle (\mu') \quad (2)$$

where  $E_n = -m \frac{C_F^2 \alpha_s^2}{4n^2}$ ,  $H_o = \frac{\mathbf{p}^2}{m} + V_o^{(0)}$  and  $|n, l\rangle$  are the Coulomb wave functions.  $V_s$  is taken at three loop leading log and the other singlet potentials in the  $1/m$  expansion at the appropriate order in  $\alpha_s$  for the requested accuracy. If  $\Lambda_{\text{QCD}} \ll mv^2$ , also the scale  $mv^2$  can be integrated out perturbatively from the above formula and, noticing that:  $m \frac{\epsilon_n n^6}{(m C_F \alpha_s)^4} = \frac{1}{3N_c} T_F \langle n, l | \mathbf{r} \frac{1}{E_n - H_o} \mathbf{r} | n, l \rangle$ , the non-perturbative contributions reduce simply to the well-known Voloshin-Leutwyler formula<sup>14</sup>

$$\delta E_{nl}^{\text{V-L}} = m \frac{\epsilon_n n^6 \pi^2 \langle F^2(0) \rangle}{(m C_F \alpha_s)^4}, \quad (3)$$

( $\epsilon_n$  is a (known) number of order 1) and thus the only nonperturbative contribution is given in this case at leading order by the local gluon condensate  $\langle F^2(0) \rangle \equiv \langle (\alpha_s/\pi) F_{\mu\nu}^a(0) F^{a\mu\nu}(0) \rangle$ . This is a well known nonperturbative parameter that can be calculated also inside QCD vacuum models<sup>4,2</sup>. Further nonperturbative corrections are still of nonpotential type and are suppressed. For quarkonium of a typical size smaller than  $1/\Lambda_{\text{QCD}}$ , the most relevant operator for the nonperturbative dynamics is the bilocal gluon condensate  $\langle \mathbf{E}^a(t) \phi(t, 0)_{ab}^{\text{adj}} \mathbf{E}^b(0) \rangle$ , which belongs to the class of the non-local non-perturbative gluon condensate  $\langle F_{\mu\nu}^a(x) \phi(x, 0)_{ab}^{\text{adj}} F_{\lambda\rho}^b(0) \rangle$ . Let me open a small

parenthesis by summarizing our knowledge of such operator and what model independent information can be obtained by the EFT.

Different parametrizations have been proposed<sup>15,16,17,18</sup> for the nonlocal gluon condensate. Because of its Lorentz structure, the correlator is in general described by two form factors. A convenient choice of these consists in the chromoelectric and chromomagnetic correlators:

$$\langle \mathbf{E}^a(x)\phi(x,0)_{ab}^{\text{adj}} \mathbf{E}^b(0) \rangle, \quad \langle \mathbf{B}^a(x)\phi(x,0)_{ab}^{\text{adj}} \mathbf{B}^b(0) \rangle.$$

The strength of the correlators is of the order of the gluon condensate. In the long range ( $x^2 \rightarrow \infty$ ) they fall off exponentially (in the Euclidean space) with some typical correlation lengths that can be measured on the lattice<sup>16,17</sup>. In<sup>16</sup> two different lengths have been obtained for the chromoelectric ( $T_g^E$ ) and the chromomagnetic correlators ( $T_g^B$ ):

$$T_g^E \neq T_g^B \simeq 0.1\text{--}0.2 \text{ fm (quenched)}. \quad (4)$$

A recent sum-rule calculation<sup>19</sup> gives  $T_g^E < T_g^B$ . The sum rule turns out not to be stable for the chromoelectric correlator, while for the chromomagnetic correlation length it gives

$$T_g^B = 0.11_{-0.02}^{+0.04} \text{ fm (quenched)}. \quad (5)$$

The interesting thing is that these  $T_g^E$  and  $T_g^B$ , which control the form of the nonperturbative correction entering the energy levels, have a precise physical interpretation. Their inverses correspond to the masses of the lowest-lying vector and pseudovector gluelumps<sup>b</sup>, respectively. This can be explicitly seen in the short-range limit,  $r = |\mathbf{x}| \rightarrow 0$ , where the hybrids operators can be explicitly constructed<sup>11,20</sup>. The suitable effective field theory is pNRQCD in the static limit. Gluelump operators are of the type  $\text{Tr}\{OH\}$ , where  $O = O^a T^a$  corresponds to a quark–antiquark state in the adjoint representation (the octet) and  $H = H^a T^a$  is a gluonic operator. By matching the QCD static hybrid operators into pNRQCD, we get the static energies (also called potentials) of the hybrids, which reduce to the gluelumps in the limit  $r \rightarrow 0$ . At leading order in the multipole expansion, they read

$$V_H(r) = V_o(r) + \frac{1}{T_g^H} + O(r^2); \quad \langle H^a(t)\phi(t,0)_{ab}^{\text{adj}} H^b(0) \rangle^{\text{non-pert.}} \simeq h e^{-it/T_g^H}. \quad (6)$$

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<sup>b</sup>The gluelump is a state formed by a  $Q\bar{Q}$  pair in the adjoint representation plus a gluon, all in the same spatial point. They are measured on the lattice and their masses are, roughly speaking, the limit of the hybrids static energies when the typical  $Q\bar{Q}$  distance  $r$  goes to zero.

Since hybrids are classified in QCD according to the representations of  $D_{\infty,h}$ , while in pNRQCD, where we have integrated out the length  $r$ , their classification is done according to the representations of  $O(3) \times C$ , the static hybrid short-range spectrum is expected to be more degenerate than the long-range one<sup>11,20</sup>. The lattice measure of the hybrid potentials done in<sup>21</sup> confirms this feature. In<sup>11</sup> it has been shown that the quantum numbers attribution of pNRQCD to the short-range operators, and the expected  $O(3) \times C$  symmetry of the effective field theory match the lattice measurements. By using only  $\mathbf{E}$  and  $\mathbf{B}$  fields and keeping only the lowest-dimensional representation we may identify the operator  $H$  for the short-range hybrids called  $\Sigma_g^{+'}$  (and  $\Pi_g$ ) with  $\mathbf{r} \cdot \mathbf{E}$  (and  $\mathbf{r} \times \mathbf{E}$ ) and the operator  $H$  for the short-range hybrids called  $\Sigma_u^-$  (and  $\Pi_u$ ) with  $\mathbf{r} \cdot \mathbf{B}$  (and  $\mathbf{r} \times \mathbf{B}$ ). Hence, the corresponding static energies for small  $r$  are

$$V_{\Sigma_g^{+'}, \Pi_g}(r) = V_o(r) + \frac{1}{T_g^E}, \quad V_{\Sigma_u^-, \Pi_u}(r) = V_o(r) + \frac{1}{T_g^B}.$$

The lattice measure of<sup>21</sup> shows that, in the short range,  $V_{\Sigma_g^{+'}, \Pi_g}(r) > V_{\Sigma_u^-, \Pi_u}(r)$ . This supports the sum-rule prediction<sup>19</sup> that the pseudovector hybrid lies lower than the vector one, i.e.  $T_g^E < T_g^B$ .

Summing up, the message is that, within the EFT, we can obtain quite a number of model-independent information on the nonperturbative contributions to the energy levels which are carried by these nonlocal condensates and we can establish interesting relations between these objects and physical 'nonperturbative' entities like the hybrids.

Let us now briefly consider the situation when  $mv > \Lambda_{\text{QCD}} > mv^2$ . The infrared sensitivity of the quark-antiquark static potential at three loops<sup>11</sup> signals that it may become sensitive to non-perturbative effects if the next relevant scale after  $mv$  is  $\Lambda_{\text{QCD}}$ . Indeed, in the situation  $mv \gg \Lambda_{\text{QCD}} \gg mv^2$ , the leading non-perturbative contribution (in  $\alpha_s$  and in the multipole expansion) to the static potential reads<sup>11</sup>

$$V_0(r)^{\text{nonpert}} = -i \frac{g^2}{N_c} T_F \frac{r^2}{3} \int_0^\infty dt e^{-itC_A\alpha_s/(2r)} \langle \mathbf{E}^a(t) \phi(t, 0)_{ab}^{\text{adj}} \mathbf{E}^b(0) \rangle (\mu'). \quad (7)$$

This term has to be summed to  $V_0^{\text{pert}}$  and it explicitly cancels the dependence of the perturbative static potential on the infrared scale  $\mu'$ . It is interesting to note that the leading contribution in the  $\Lambda_{\text{QCD}}/mv^2$  expansion of  $V^{\text{non-pert}}$  (obtained by putting the exponential equal to 1) cancels the order  $\Lambda_{\text{QCD}}^3 r^2$  renormalon that affects the static potential (the leading-order renormalon, of order  $\Lambda_{\text{QCD}}$ , cancels against the pole mass). Therefore, also in the renormalon

language, the above operator is the relevant non-perturbative contribution to the static potential in the considered kinematic situation.

The nonperturbative terms in the potential are in this case organized in powers of  $\alpha_s/(r\Lambda_{\text{QCD}})$ , starting with a quadratic term,  $r^2$ . The actual form of these short range nonperturbative corrections to the potential depends on the actual form of the nonlocal correlator<sup>11</sup>. It is therefore very interesting to calculate this object inside models of the QCD vacuum, or with topological configurations on the lattice, also in relation to the recent claim of violation of the OPE expansion 'carried' by a nonperturbative short distance 'string' term in the static singlet potential<sup>2</sup>.

#### 4 Nonrelativistic quark bound states with a large characteristic radius: Wilson loops

Fig. 1 shows that most of the quarkonia states lie in a region where the inverse of the size of the system is close to the scale  $\Lambda_{\text{QCD}}$ . In this situation the potential can no longer be expressed as an expansion in  $\alpha_s$ . The non-perturbative dynamics is already switched on at the binding (potential) scale and it is contained in more extended objects than (local or non-local) gluon condensates: Wilson loops and chromoelectric and chromomagnetic field insertions on the Wilson loops. Such operators can be eventually calculated on the lattice<sup>23</sup> or in QCD vacuum models<sup>24,1</sup>. In<sup>22</sup> the matching of QCD to pNRQCD has been performed at order  $1/m^2$  in the general situation  $\Lambda_{\text{QCD}} \lesssim mv$ . This has been proved to be equivalent to compute the heavy quarkonium potential at order  $1/m^2$ . More precisely, a pure potential picture emerges at the leading order in the ultrasoft expansion under the condition that all the gluonic excitations (hybrids) have a gap of  $O(\Lambda_{\text{QCD}})$  with respect to the singlet. Such situation is confirmed by lattice simulations<sup>23</sup>. Then if we switch off the light fermions (pure gluodynamics), only the singlet survives and pNRQCD reduces to a pure two-particle NR quantum-mechanical system. Therefore, the situation *assumed* by all potential models, may be now rigorously *derived* under a specific set of circumstances (for more details see<sup>22</sup>). In this situation, pNRQCD involves only a bilinear colour singlet field,  $S(\mathbf{x}_1, \mathbf{x}_2, t)$ ,

$$\mathcal{L}_{\text{pNRQCD}} = S^\dagger \left( i\partial_0 - h_s(\mathbf{x}_1, \mathbf{x}_2, \mathbf{p}_1, \mathbf{p}_2) \right) S, \quad (8)$$

where  $h_s$  is the Hamiltonian of the singlet,  $\mathbf{p}_1 = -i\nabla_{\mathbf{x}_1}$  and  $\mathbf{p}_2 = -i\nabla_{\mathbf{x}_2}$ . It has the following expansion up to order  $1/m^2$

$$h_s = \frac{\mathbf{p}_1^2}{2m_1} + \frac{\mathbf{p}_2^2}{2m_2} + V^{(0)} + \frac{V^{(1,0)}}{m_1} + \frac{V^{(0,1)}}{m_2} + \frac{V^{(2,0)}}{m_1^2} + \frac{V^{(0,2)}}{m_2^2} + \frac{V^{(1,1)}}{m_1 m_2}. \quad (9)$$



Higher order effects in the  $1/m$  expansion as well as extra ultrasoft degrees of freedom<sup>11,22,25</sup>, such as hybrids and pions can be systematically included and may eventually affect the leading potential picture (like in the perturbative regime ultrasoft gluons<sup>11,22</sup>). We shall use the following notations:  $\langle \dots \rangle$  will stand for the average over the Yang–Mills action,  $W_\square$  for the rectangular static Wilson loop of dimensions  $r \times T$  and  $\langle\langle \dots \rangle\rangle \equiv \langle \dots W_\square \rangle / \langle W_\square \rangle$ . We define  $\langle\langle O_1(t_1) \dots O_n(t_n) \rangle\rangle_c$  as the *connected* Wilson loop. We get in terms of Wilson loops<sup>22</sup>

$$V^{(0)}(r) = \lim_{T \rightarrow \infty} \frac{i}{T} \ln \langle W_\square \rangle, \quad (10)$$

$$V^{(1,0)}(r) = -\frac{1}{2} \lim_{T \rightarrow \infty} \int_0^T dt \, t \, \langle\langle g\mathbf{E}_1(t, \mathbf{x}_1) \cdot g\mathbf{E}_1(0, \mathbf{x}_1) \rangle\rangle_c = V^{(0,1)}(r). \quad (11)$$

All the obtained expression for the potentials are manifestly gauge invariant and are also correct at any power in  $\alpha_s$  in the perturbative regime. For the terms of order  $1/m^2$  we have the general decomposition

$$V^{(2,0)} = \frac{1}{2} \left\{ \mathbf{p}_1^2, V_{\mathbf{p}^2}^{(2,0)}(r) \right\} + \frac{V_{\mathbf{L}^2}^{(2,0)}(r)}{r^2} \mathbf{L}_1^2 + V_r^{(2,0)}(r) + V_{LS}^{(2,0)}(r) \mathbf{L}_1 \cdot \mathbf{S}_1, \quad (12)$$

where  $\mathbf{L}_j \equiv \mathbf{r} \times \mathbf{p}_j$ . Similar definitions hold for  $V^{(0,2)}$  and  $V^{(1,1)}$ . All the potentials (i.e. the functions of  $r$ ) in  $V^{(2,0)}, V^{(0,2)}, V^{(1,1)}$  are obtained factorized in two contributions. The first one contains the hard physics at the scale  $m$  which is calculated as a series expansion in  $\alpha_s$  at the appropriate hard scale<sup>22,10</sup>. The second contribution at the scale  $1/r$ , enjoys a closed expression in terms of average values of two or more chromoelectric and chromomagnetic insertions in the presence of the Wilson loop<sup>22</sup>, similarly to (10),(11). The explicit expressions can be found in<sup>22</sup>. *Therefore, the calculation of the dynamics, i.e. the calculation of the potential, is in this way reduced to a calculation in pure gluodynamics at the lowest scale.* On one hand, such objects are very conveniently evaluated on the lattice since they are pure glue objects and contain only one scale. On the other hand, such objects can be very well evaluated using nontrivial topological configurations inside a model of the QCD vacuum<sup>24,1</sup>. We emphasize that, since the potential we get here is a well defined quantity, derived from QCD via a systematic and unambiguous procedure, and *complete* up to order  $1/m^2$ , it is not affected by the usual ambiguities (ordering, retardation corrections, etc.), which affect all potential models and all phenomenological reductions of Bethe–Salpeter kernels<sup>9</sup>. For the same reason the above results are the relevant ones for the study of the properties of the QCD vacuum in the presence of heavy sources. The average value of the field insertions in the presence of the Wilson loop contain all the relevant

information about the heavy quark dynamics. This is much more detailed and quantitative with respect to the generic request that topological configuration should originate an area law behaviour into the Wilson loop. If we calculate these field insertions on the Wilson loop using some dominant topological configurations we have then a simple procedure to relate the result, on one hand to standard lattice simulations of the same objects and, on the other hand, directly to the spectrum. This last feature descends from the fact that within the EFT we are able to fully take into account the perturbative contributions and we have a clear power counting that selects the terms contributing at a given order in the  $v$  expansion.

## 5 Topology and heavy quark bound states

The outlined study is rigorous and allows a systematic disentanglement of the high from the low energy scales of the heavy quarkonium system under study. In this specific sense also perturbative and nonperturbative effects turn out to be disentangled. Thus the nonrelativistic dynamics of heavy quark bound states is surely an adequate benchmark to test ideas and models of QCD topology and QCD vacuum structure.

As I have shown, when  $mv \sim \Lambda_{\text{QCD}}$ , then the whole nonperturbative  $Q\bar{Q}$  interaction at order  $1/m^2$  turns out to be expressed in terms of average values of Wilson loops and field insertions into the Wilson loop. We know that in the confined phase the Wilson loop is dominated by an area law<sup>9</sup> which is related to the formation of an interquark confining flux tube<sup>27</sup>. Such feature has been obtained using different topological configurations: gluon Abelian projected fields<sup>2</sup>, monopoles<sup>2</sup>, center vortices<sup>5</sup>. In all these cases an approximated area law is obtained and it is debated what are the relevant topological configurations that really drive the confinement dynamics. Many lattice investigations have been performed and have ended up with the notion that in a way all these topological configurations are related<sup>26</sup>. Several QCD vacuum models exist in the literature<sup>15,1,24,9</sup> and all of them obtain an area law for the Wilson loop. However, there are quite a number of differences between different vacuum models as well as between different topological configurations, but such differences are not apparent at the level of the static Wilson loop. In this case, typically we need only one parameter, the string tension  $\sigma$ , to encode the information of a constant energy density in the confining flux tube. However, there are many ways in which this can be realized in relation to the actual profile of the flux tube. It is well known for example that while the  $Q\bar{Q}$  flux tube measured on the lattice in QCD is quite fat, the one measured on the lattice in the Abelian projection is very thin<sup>28</sup>.

On the other hand in order to describe the nonperturbative heavy quark dynamics at order  $1/m^2$  we need not only the Wilson loop area law behaviour but also the behaviour of the chromoelectric and chromomagnetic field insertion into the Wilson loop. These last contain information on the profile of the flux tube and turn out to be much more sensitive to the QCD vacuum models or the topological configurations<sup>9,24</sup> <sup>c</sup>. Thus, it is much more interesting to study the full non relativistic dynamics of heavy quarks in place of the much more qualitative statement about finding out confining configurations into the Wilson loop. In particular, it would be interesting to have a lattice calculations of (10), (11), (12) and the other  $1/m^2$  potentials contained in <sup>22</sup>, using monopoles configurations<sup>2</sup> versus center vortices<sup>4,5</sup>. In conclusion, I would like to invite the people working in the field to calculate local gluon condensates, nonlocal gluon condensates, Wilson loops and field insertions into the Wilson loop in their topological models and to relate them to the phenomenological data using the unified and systematic frame of the EFT that I presented here.

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<sup>c</sup>In two models for the QCD vacuum, stochastic vacuum model<sup>15</sup> and Dual QCD<sup>1</sup>, the profile of the flux tube is controlled by the correlation length  $T_g$  given in equation (4) and thus it is ultimately related to hybrids configurations.

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